

CHAPTER 14 다변수함수

경영·경제분석을 위한 핵심 수학(6판)

연습문제 풀이 이용 안내

- 본 문제 풀이의 저작권은 원서 저작권자(Knut Sydsæter, Arne Strøm, Peter Hammond, Andrés Carvajal)와 Pearson, 한빛아카데미(주)에 있습니다.
- 이 자료를 무단으로 전제하거나 배포할 경우 저작권법 136조에 의거하여 최고 5년 이하의 징역 또는 5천만원 이하의 벌금에 처할 수 있고 이를 병과(併科)할 수도 있습니다.

CHAPTER 14 다변수함수

14.1 이변수함수

1.

$$f(0,1)=2, f(2,-1)=0, f(a,a)=3a, f(a+h,b)-f(a,b)=h$$

2.

$$f(0,1)=0, f(-1,2)=-4, f(10^4, 10^{-2})=1, f(a,a)=a^3, f(a+h,b)-f(a,b)=hb^2, f(a,b+k)-f(a,b)=2abk+ak^2$$

3.

$$f(1,1)=2, f(-2,3)=51, f\left(\frac{1}{x}, \frac{1}{y}\right)=\frac{3}{x^2}-\frac{2}{xy}+\frac{1}{y^3} \quad p=6x+3h-2y, q=-2x+3y^2+3yk+k^2$$

4.

$$(a) f(-1,2)=1, f(a,a)=4a^2, f(a+h,b)-f(a,b)=2(a+b)h+h^2$$

(b) 증명 생략

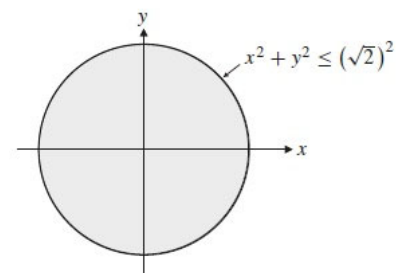
5.

$$F(1,1)=10, F(4,27)=60, F\left(9, \frac{1}{27}\right)=10, F(3, \sqrt{2})=10\sqrt{3}, F(100,1000)=1000, F(2K,2L)=2^{5/6}F(K,L)$$

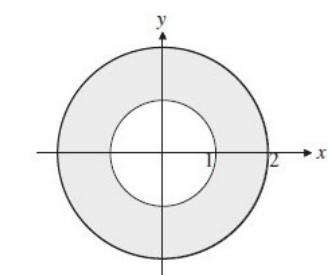
6.

$$(a) y \neq x-2$$

$$(b) x^2+y^2 \leq 2$$



$$(c) 1 \leq x^2+y^2 \leq 4$$



7.

$$(a) x+y \neq \ln 3$$

$$(b) x \neq a, y \neq b$$

$$(c) x > a, y > b$$

CHAPTER 14 다변수함수

14.2 이변수함수의 편도함수

1.

- (a) $\frac{\partial z}{\partial x} = 2, \frac{\partial z}{\partial y} = 3$ (b) $\frac{\partial z}{\partial x} = 2x, \frac{\partial z}{\partial y} = 3y^2$
 (c) $\frac{\partial z}{\partial x} = 20x^3y^4, \frac{\partial z}{\partial y} = 4x^3y^3$ (d) $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} = 2(x+y)$

2.

- (a) $\frac{\partial z}{\partial x} = 2x, \frac{\partial z}{\partial y} = 6y$
 (b) $\frac{\partial z}{\partial x} = y, \frac{\partial z}{\partial y} = x$
 (c) $\frac{\partial z}{\partial x} = 20x^3y^2 - 2y^5, \frac{\partial z}{\partial y} = 10x^4y - 10xy^4$
 (d) $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} = e^{x+y}$
 (e) $\frac{\partial z}{\partial x} = ye^{xy}, \frac{\partial z}{\partial y} = xe^{xy}$
 (f) $\frac{\partial z}{\partial x} = \frac{e^x}{y}, \frac{\partial z}{\partial y} = -\frac{e^x}{y^2}$
 (g) $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} = \frac{1}{x+y}$
 (h) $\frac{\partial z}{\partial x} = \frac{1}{x}, \frac{\partial z}{\partial y} = \frac{1}{y}$

3.

- (a) $f'_1(x, y) = 7x^6, f'_2(x, y) = -7y^6, f''_{12}(x, y) = 0$
 (b) $f'_1(x, y) = 5x^4 \ln y, f'_2(x, y) = \frac{x^5}{y}, f''_{12}(x, y) = \frac{5x^4}{y}$
 (c) $f'_1(x, y) = 10x(x^2 - 2y^2)^4, f'_2(x, y) = -20y(x^2 - 2y^2)^4, f''_{12}(x, y) = -160xy(x^2 - 2y^2)^3$

4.

- (a) $z'_x = 3, z'_y = 4, z''_{xx} = z''_{xy} = z''_{yx} = z''_{yy} = 0$
 (b) $z'_x = 3x^2y^2, z'_y = 2x^3y, z''_{xx} = 6xy^2, z''_{yy} = 2x^3, z''_{xy} = 6x^2y$
 (c) $z'_x = 5x^4 - 6xy, z'_y = -3x^2 + 6y^5, z''_{xx} = 20x^3 - 6y, z''_{yy} = 30y^4, z''_{xy} = -6x$
 (d) $z'_x = \frac{1}{y}, z'_y = -\frac{x}{y^2}, z''_{xx} = 0, z''_{yy} = \frac{2x}{y^3}, z''_{xy} = -\frac{1}{y^2}$
 (e) $z'_x = 2y(x+y)^{-2}, z'_y = -2x(x+y)^{-2}, z''_{xx} = -4y(x+y)^{-3}, z''_{yy} = 4x(x+y)^{-3}, z''_{xy} = 2(x-y)(x+y)^{-3}$
 (f) $z'_x = x(x^2 + y^2)^{-1/2}, z'_y = y(x^2 + y^2)^{-1/2}, z''_{xx} = y^2(x^2 + y^2)^{-3/2}, z''_{yy} = x^2(x^2 + y^2)^{-3/2}, z''_{xy} = -xy(x^2 + y^2)^{-3/2}$

5.

- (a) $z'_x = 2x, z'_y = 2e^{2y}, z''_{xx} = 2, z''_{yy} = 4e^{2y}, z''_{xy} = 0$
 (b) $z'_x = \frac{y}{x}, z'_y = \ln x, z''_{xx} = -\frac{y}{x^2}, z''_{yy} = 0, z''_{xy} = \frac{1}{x}$

CHAPTER 14 다변수함수

(c) $z'_x = y^2 - ye^{xy}$, $z'_y = 2xy - xe^{xy}$, $z''_{xx} = -y^2e^{xy}$, $z''_{yy} = 2x - x^2e^{xy}$, $z''_{xy} = 2y - e^{xy} - xye^{xy}$

(d) $z'_x = yx^{y-1}$, $z'_y = x^y \ln x$, $z''_{xx} = y(y-1)x^{y-2}$, $z''_{yy} = x^y(\ln x)^2$, $z''_{xy} = x^{y-1} + yx^{y-1} \ln x$

6.

(a) $F'_S(S, E) = 0.9944S^{-0.56}E^{0.48}$, $F'_E(S, E) = 1.0848S^{0.44}E^{-0.52}$

(b) 증명 생략

7.

증명 생략

8.

증명 생략

9.

(a) $s'_x(20, 30) = \frac{1}{10}$

(b) $s'_y(20, 30) = \frac{2}{15}$

CHAPTER 14 다변수함수

14.3 기하적 표현

1.

증명 생략

2.

증명 생략

3.

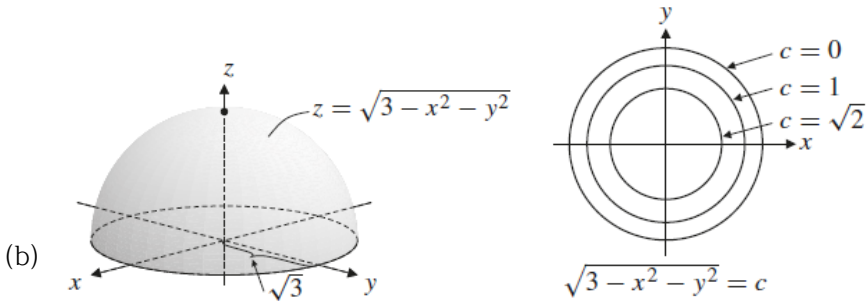
증명 생략

4.

증명 생략

5.

(a) 생략



6.

(a) $f(2,3)=8$

$$x=2, x=5$$

(b) $y=3$ 일 때 $f(2,y)=8$ 이 최솟값

(c) A 에서 $f'_1(x,y)>0, f'_2(x,y)>0$

$$B\text{에서 } f'_1(x,y)<0, f'_2(x,y)<0$$

$$C\text{에서 } f'_1(x,y)=0, f'_2(x,y)=0$$

$$f'_1 \approx 2, f'_2 \approx \frac{10}{3}$$

7.

(a) 점 P 에서 $f'_x > 0, f'_y < 0$

점 Q 에서 $f'_x < 0, f'_y > 0$

(b) (i) 이 그림에서 해를 구할 수 없다.

$$(ii) x \approx 2, x \approx 6$$

(c) 3

8.

$$F(1,0)-F(0,0) \geq 2$$

$$F(2,0)-F(1,0) \geq 2$$

$$F(1,1)-F(0,1) \geq 2$$

$$F(0,1)-F(0,0) \leq 1$$

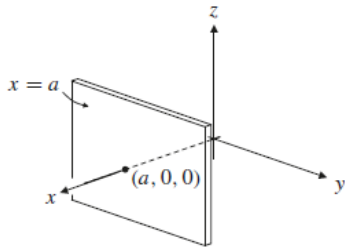
$$F(1,1)-F(1,0) \leq 1$$

CHAPTER 14 다변수함수

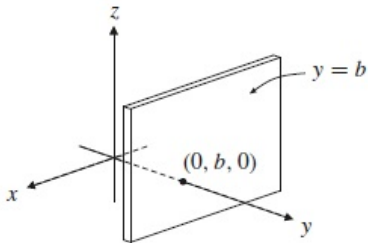
14.4 곡면과 거리

1.

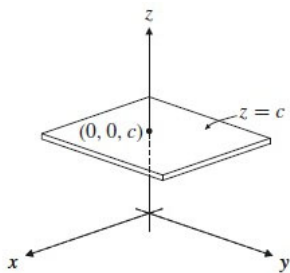
(a)



(b)



(c)



2.

(a) $5\sqrt{2}$

(b) $\sqrt{3}$

3.

$$(x-2)^2 + (y-2)^2 + (z-1)^2 = 25$$

4.

중심이 $(-3, 3, 4)$ 이고 반지름이 5인 구

5.

점 $\left(4, 4, \frac{1}{2}\right)$ 에서 (x, y, z) 까지 거리의 제곱

CHAPTER 14 다변수함수

14.5 n 변수함수

1.

(a) $f(-1, 2, 3) = 1, f(a+1, b+1, c+1) - f(a, b, c) = 2a + 2b + 2c + 3$

(b) 증명 생략

2.

(a) $2^{1.053} \approx 2.07$ 배로 변한다.

(b) $\ln y = \ln 2.9 + 0.015 \ln x_1 + 0.25 \ln x_2 + 0.35 \ln x_3 + 0.408 \ln x_4 + 0.03 \ln x_5$

3.

(a) 약 13.167백만 주

(b) 조화평균

4.

(a) 총 $e = \sum_{w=1}^n \frac{100}{p_w}$ 백만 유로

(b) $\frac{1}{n} \sum_{w=1}^n \frac{1}{p_w}$ 달러

5.

(a) 8분

(b) 증명 생략

CHAPTER 14 다변수함수

14.6 다변수함수의 편도함수

1.

$$F'_1(1,1,1)=4e, F'_2(1,1,1)=4e, F'_3(1,1,1)=e$$

2.

$$(a) f'_1=2x, f'_2=3y^2, f'_3=4z^3$$

$$(b) f'_1=10x, f'_2=-9y^2, f'_3=12z^3$$

$$(c) f'_1=yz, f'_2=xz, f'_3=xy$$

$$(d) f'_1=\frac{4x^3}{yz}, f'_2=-\frac{x^4}{y^2z}, f'_3=-\frac{x^4}{yz^2}$$

$$(e) f'_1=12x(x^2+y^3+z^4)^5, f'_2=18y^2(x^2+y^3+z^4)^5, f'_3=24z^3(x^2+y^3+z^4)^5$$

$$(f) f'_1=yz e^{xyz}, f'_2=xz e^{xyz}, f'_3=xy e^{xyz}$$

3.

$$\frac{\partial T}{\partial x} > 0, \frac{\partial T}{\partial y} > 0, \frac{\partial T}{\partial d} < 0$$

4.

$$(a) g(2,1,1)=-2, g(3,-4,2)=352, g(1,1,a+h)-g(1,1,a)=2ah+h^2-h$$

$$(b) g'_1=4x-4y-4, g'_2=-4x+20y-28, g'_3=2z-1$$

$$g''_{11}=4, g''_{12}=-4, g''_{13}=0, g''_{21}=-4, g''_{22}=20, g''_{23}=0, g''_{31}=0, g''_{32}=0, g''_{33}=2$$

5.

$$\frac{\partial \pi}{\partial p} = \frac{1}{2}p\left(\frac{1}{r} + \frac{1}{w}\right), \frac{\partial \pi}{\partial r} = -\frac{1}{4}\frac{p^2}{r^2}, \frac{\partial \pi}{\partial w} = -\frac{1}{4}\frac{p^2}{w^2}$$

6.

$$w'_1=3yz+2xy-z^3, w'_2=3xz+x^2, w'_3=3xy-3xz^2$$

$$w''_{11}=2y, w''_{12}=w''_{21}=3z+3x, w''_{13}=w''_{31}=3y-3z^2, w''_{22}=0, w''_{23}=w''_{32}=3x, w''_{33}=-6xz$$

7.

$$f'_1=p'(x), f'_2=q'(y), f'_3=r'(z)$$

8.

$$(a) \begin{pmatrix} 2a & 0 & 0 \\ 0 & 2b & 0 \\ 0 & 0 & 2c \end{pmatrix} \quad (b) \begin{pmatrix} \frac{a(a-1)g}{x^2} & \frac{abg}{xy} & \frac{acg}{xz} \\ \frac{abg}{xy} & \frac{b(b-1)g}{y^2} & \frac{bcg}{yz} \\ \frac{acg}{xz} & \frac{bcg}{yz} & \frac{c(c-1)g}{z^2} \end{pmatrix}$$

9.

증명 생략

10.

$$f'_x=y^z x^{y^z-1}, f'_y=zy^{z-1}(\ln x)x^{y^z}, f'_z=y^z(\ln x)(\ln y)x^{y^z}$$

11.

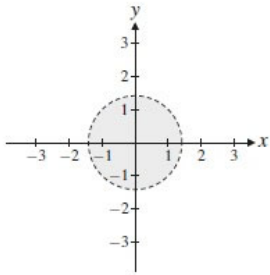
$$f'_1(0,y)=-y, f'_2(x,0)=x$$

CHAPTER 14 다변수함수

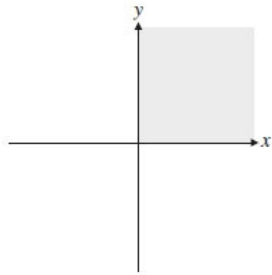
14.7 볼록집합

1.

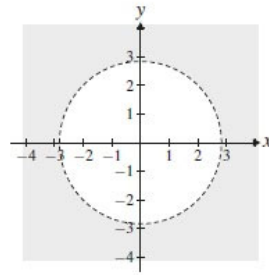
(a)



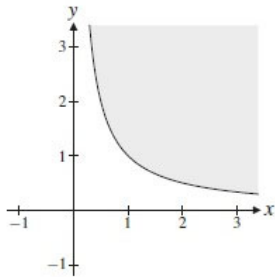
(b)



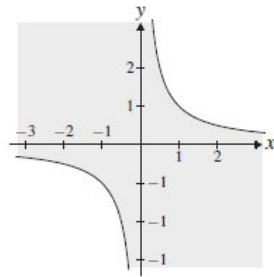
(c)



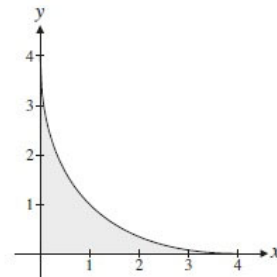
(d)



(e)



(f)



2.

증명 생략

CHAPTER 14 다변수함수

14.8 오목함수와 볼록함수

1.

$f(x, y)$: 순볼록

$g(x, y)$: 볼록도 아니고 오목도 아님

$h(x, y)$: 오목하나 순오목하지 않음

2.

$-2 - \sqrt{6} \leq a \leq -2 + \sqrt{6}$ 이면 오목

3.

(a) 순오목

(b) 순볼록

(c) 오목하나 순오목하지 않음

4.

증명 생략

CHAPTER 14 다변수함수

14.9 경제학에의 응용

1.

$$\frac{\partial M}{\partial Y} = 0.14 > 0, \frac{\partial M}{\partial r} = -63.8652(r-2)^{-1.84} < 0$$

2.

(a) aY

(b) $(a+b)Y$

(c) Y

3.

$$D'_p(p, q) = -bq^{-\alpha} < 0, D'_q(p, q) = bp\alpha q^{-\alpha-1} > 0$$

4.

증명 생략

5.

$$\frac{\partial D}{\partial p} < 0, \frac{\partial E}{\partial q} < 0, \frac{\partial D}{\partial q} > 0, \frac{\partial E}{\partial p} > 0$$

6.

$$i = 1, \dots, n \text{에 대해} \quad \frac{\partial U}{\partial x_i} = e^{-x_i}$$

7.

$$\mu Y$$

CHAPTER 14 다변수함수

14.10 부분탄력성

1.

(a) $\text{El}_x z = 1, \text{El}_y z = 1$

(b) $\text{El}_x z = 2, \text{El}_y z = 5$

(c) $\text{El}_x z = n + x, \text{El}_y z = x + y$

(d) $\text{El}_x z = \frac{x}{x+y}, \text{El}_y z = \frac{y}{x+y}$

2.

$$\sum_{i=1}^3 \text{El}_i z = \frac{adgx_1^d}{ax_1^d + bx_2^d + cx_3^d} + \frac{bdgx_2^d}{ax_1^d + bx_2^d + cx_3^d} + \frac{cdgx_3^d}{ax_1^d + bx_2^d + cx_3^d} = dg$$

3.

$i = 1, \dots, n$ 에 대해 $\text{El}_i z = p + a_i x_i$

4.

증명 생략

CHAPTER 14 다변수함수

CHAPTER 14 복습문제

14.1

$$f(0,1)=-5, f(2,-1)=11, f(a,a)=-2a, f(a+h,b)-f(a,b)=3h$$

14.2

$$f(-1,2)=-10, f(2a,2a)=-4a^2, f(a,b+k)-f(a,b)=-6bk-3k^2, f(tx,ty)-t^2f(x,y)=0$$

14.3

$$f(3,4,0)=5, f(-2,1,3)=\sqrt{14}, f(tx,ty,tz)=\sqrt{t^2x^2+t^2y^2+t^2z^2}=tf(x,y,z)$$

14.4

$$(a) F(0,0)=0, F(1,1)=15, F(32,243)=270$$

$$(b) F(K+1,L)-F(K,L)=15L^{2/5}[(K+1)^{1/5}-K^{1/5}]$$

$$(c) F(32+1,243)-F(32,243)\approx 1.667, F'_K(32,243)=\frac{27}{16}\approx 1.6875$$

(d) 증명 생략

14.5

$$(a) \frac{\partial Y}{\partial K}\approx 0.083K^{-0.356}S^{0.562}, \frac{\partial Y}{\partial S}\approx 0.035K^{-1.356}S^{-0.438}$$

$$(b) 2^{1.918}\approx 3.779배$$

14.6

$$(a) \text{ 모든 } (x,y)$$

$$(b) xy\leq 1$$

$$(c) x^2+y^2<2$$

14.7

$$(a) x+y>1$$

$$(b) x^2+y^2\geq 1, |x|\geq |y|$$

$$(c) 0\leq x\leq 1, \sqrt{x}\geq y\geq x^2$$

14.8

$$(a) 10xy^2(x^2y^4+2)^4$$

$$(b) \sqrt{K}+\sqrt{L}$$

$$(c) KF'_K(K,L)+LF'_L(K,L)=F$$

$$(d) -\frac{3}{w^2}+2t$$

$$(e) t_3(t_1^2+t_2^2+t_3^2)^{-1/2}$$

$$(f) f'_1(x,y,z)=4xyz+2xz^2, f''_{13}(x,y,z)=4xy+4xz$$

14.9

$$(a) f(0,0)=36, f(-2,-3)=0, f(a+2,b-3)=a^2b^2$$

$$(b) f'_x=2(x-2)(y+3)^2, f'_y=2(x-2)^2(y+3)$$

14.10

증명 생략

CHAPTER 14 다변수함수

14.11

증명 생략

14.12

(a) $f'_1(a, y) = 4x^3 - 8xy, f'_2(x, y) = 4y - 4x^2 + 4$

(b) $(0, -1), (\sqrt{2}, 1), (-\sqrt{2}, 1)$

14.13

(a) 오목하지도 볼록하지도 않다.

(b) 증명 생략

14.14

증명 생략

14.15

(a) 증명 생략

(b) f 가 오목 $\Leftrightarrow a \leq 0, c \leq 0, ac - b^2 \geq 0$

f 가 볼록 $\Leftrightarrow a \geq 0, c \geq 0, ac - b^2 \geq 0$

14.16

(a) $\text{El}_x z = 3, \text{El}_y z = -4$

(b) $\text{El}_x z = \frac{2x^2}{(x^2 + y^2)\ln(x^2 + y^2)}, \text{El}_y z = -\frac{2y^2}{(x^2 + y^2)\ln(x^2 + y^2)}$

(c) $\text{El}_x z = x, \text{El}_y z = y$

(d) $\text{El}_x z = \frac{x^2}{x^2 + y^2}, \text{El}_y z = \frac{y^2}{x^2 + y^2}$

14.17

(a) $\frac{\partial F}{\partial y} = -2e^{2x}(1 - y)$

(b) $F'_L = \frac{(\ln K)(\ln M)}{L}, F'_{LK} = \frac{\ln M}{KL}$

(c) $w'_x = x^x y^x z^x [\ln(xyz) + 1]$

14.18

(a) $(-1)q - 1(q - 1)!$

(b) $(p + 1)q - 1$

14.19

증명 생략